Parameters Extraction and Modeling for Planar Spiral Inductor on Si–SiO$_2$ Substrates by DDM for Conformal Modules

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***Abstract*—In this paper, the domain decomposition method (DDM) for conformal modules is used to get simple analytic expressions for parameters of planar spiral inductors on Si–SiO$_2$ substrates. The conductor and substrate losses are considered in the expressions. The quality factor of the spiral inductor is computed with a transmission-line mode and compared with previously published experimental results, showing that DDM model is accurate and efficient for modeling an on-chip spiral inductor on Si–SiO$_2$ substrates.**

***Index Terms*—Conformal module, domain decomposition method (DDM), planar spiral inductors, quality factor.**

I. INTRODUCTION

MONOLITHIC inductors, especially in the form of spirals, have gained much application in the design of integrated RF transmitters and receivers. For this reason, the analysis and optimization of such structures have been of great importance. Considerable research work has been done over the past several years [1]–[4]. These methods are based on numerical techniques, empirical formulate, and physical models. However, the electrical parameters of spiral inductors in these methods lack a simple and accurate formula. This is a major impediment in using these methods for quick design and optimization of spiral inductors.

In this paper, the domain decomposition method (DDM) for conformal modules is used to get simple analytic expressions for parameters of planar spiral inductors on Si–SiO$_2$ substrates. The DDM for conformal modules can avoid complex conformal mapping. It transforms directly a planar spiral to an elongated rectangular strip. As a result, the complex problem of computing the parameters of planar spiral inductors is transformed to computing the parameters of the simple rectangular strip. By using the DDM for conformal modules, the expressions of a quality factor for a plane spiral inductor are obtained. The computed results from the expressions are compared with previously published experimental results and good agreement is achieved.

Manuscript received October 23, 2002; revised January 21, 2003. This work was supported by the National Science Foundation for Outstanding Youth of China under Grant 60025103, by the National High Technology Research and Development Program of China under Grant 2002AA1Z1520, and by the Shanghai AM Foundation under Grant 0110.

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Digital Object Identifier 10.1109/TMTT.2003.812581

II. IMPEDANCE AND QUALITY FACTOR WITH DDM

The concept of the conformal module of a quadrilateral is introduced first. Let $Q := \{\Omega; z_1, z_2, z_3, z_4\}$ denote a quadrilateral consisting of a Jordan domain Ω and four specified points z_1, z_2, z_3, and z_4 on boundary $\partial\Omega$. The conformal module $m(Q)$ of Q is defined as follows. Suppose R_h denotes a rectangle of the form $R_h := \{(\xi, \eta) : 0 < \xi < 1, 0 < \eta < h\}$, and then $m(Q)$ is the unique value of h for which Q is conformally equivalent to the rectangular quadrilateral $\{R_h; 0, 1, 1+ih, ih\}$. By this, we mean that, for $h = m(Q)$, and for this value only, there exits a unique conformal map $F : \Omega \rightarrow R_h$, which takes the four points z_1, z_2, z_3, and z_4, respectively, onto the four vertices 0, 1, $1+ih$, and ih of R_h [5], [6].

Now consider the plane spiral inductors on a lossy Si substrate with a thin SiO$_2$ dielectric film between the signal conductor and Si substrate, as shown in Fig. 1(a) and (b). Both the SiO$_2$ film and Si substrate are uniform and infinite in the horizontal plan. The structure parameters of the spiral inductor are width w, spacing width s, thickness t_m, and the total length l. The thickness of SiO$_2$ and Si is $t_{\rm ox}$ and $t_{\rm Si}$, respectively. The signal conductor is considered as a quadrilateral Q and is divided into n domains Q_i, $i = 1, \ldots, n$, as described in Fig. 1(a) and (b). The crosscuts of subdivisions for the decomposition are chosen such as that in [5]. According to the DDM, the total conformal module of Q is approximated by

$$m(Q) = \sum_{i=1}^{n} m(Q_i). \quad (1)$$

Let T_l and A denote the trapezium and half- circular annulus sub-domain in Fig. 1(a) and (b), respectively (see Fig. 2). The conformal $m(T_l)$ and $m(A)$ are then expressed as follows [5], [7]:

$$m(T_l) = \frac{2l - w}{2w} - \frac{1}{\pi}\log 2, \qquad \text{for } l \gg w \quad (2)$$

$$m(A) = \frac{\pi}{\log R - \log r}. \quad (3)$$

Subdivisions Q_2–Q_{n-1} in Fig. 1(a) are symmetrical trapeziums, which can be divided into two equal quadrilaterals like T_l in Fig. 2. Thus, by substituting the geometrical size of each subdivision in Fig. 1(a) into (2) and using (1), we get the approximated total module of the square spiral inductor, which is

$$m^s(Q) = \frac{4N(d_{\rm out} - w)}{w} + \frac{(1 + 2N - 8N^2)(s + w)}{2w}$$

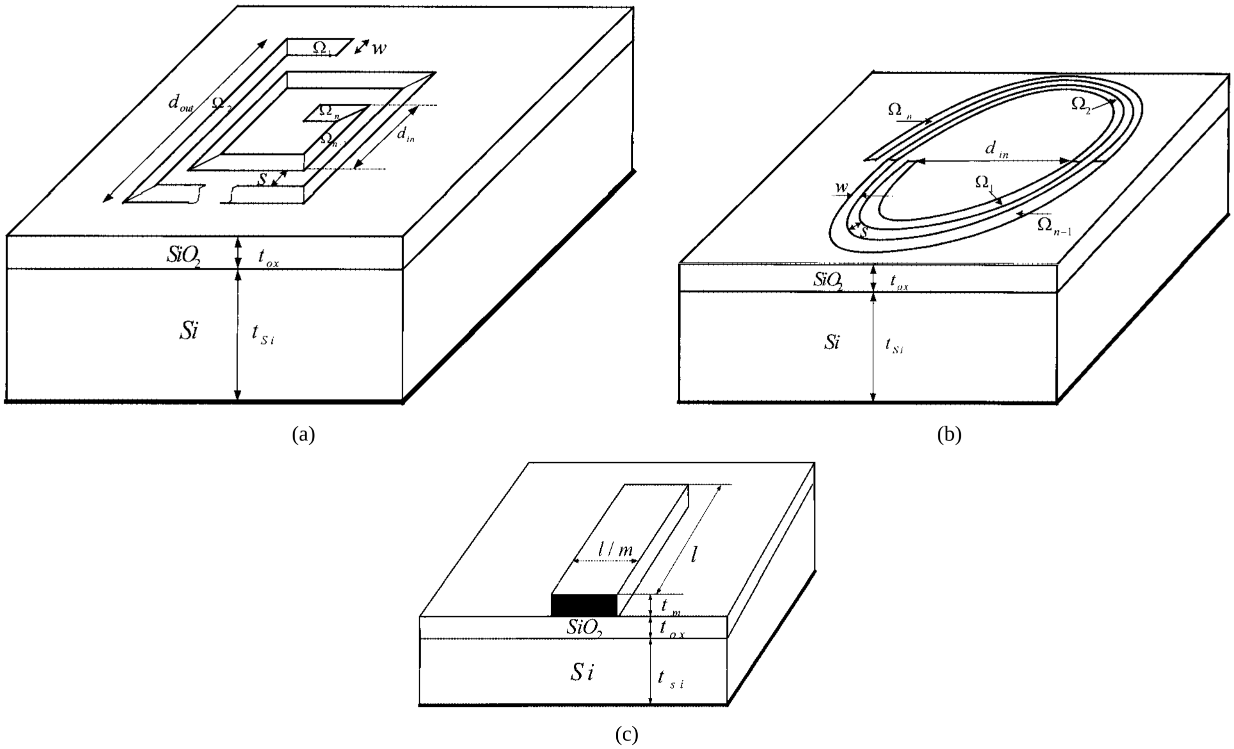

Fig. 1. On-chip inductor and its transformation by the DDM for conformal modules. (a) Square spiral inductor on Si–SiO_2 substrate. (b) Circular spiral inductor on Si–SiO_2 substrate. (c) Microstrip structure obtained from (a) and (b) by DDM for conformal modules.

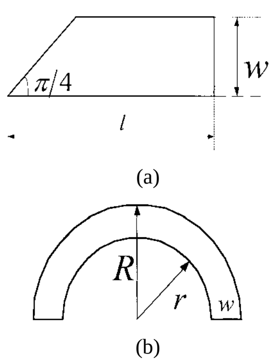

Fig. 2. (a) Trapezium sub-domain. (b) Half-circular annulus sub-domain.

$$-\frac{8N}{\pi}\log 2 \tag{4}$$

$$d_{\text{out}} = d_{\text{in}} + (2N-1)(s+w). \tag{5}$$

Similar for the circular spiral inductor, we have

$$m^c(Q) = \pi \sum_{i=0}^{2N-1} \frac{1}{\ln\left(1+\dfrac{2w}{d_{\text{in}}+i(w+s)}\right)} \tag{6}$$

where N is the turns of the inductor, d_{out} is the outer dimension, and d_{in} is the inner dimension.

The error of $m(Q)$ is [6]

$$E = 4.41\Big[\exp\big(-2\pi m(Q_1)\big) + \sum_{j=3}^{n}\exp\big(-2\pi m(Q_j)\big)\Big]. \tag{7}$$

By the DDM, the spiral inductors in Fig. 1(a) and (b) are now transformed to the microstrip structure in Fig. 1(c). According to the method that conformal mapping extracts parameters [8], we can determine that the stripline length is l and the stripline width is $w_{\text{eq}} = l/m$. In the entire process, the parameters t_m, t_{ox}, and t_{Si} keep constant and we assume $t_{\text{Si}} \gg w$, $t_{\text{Si}} \gg t_m$, and $t_{\text{Si}} \gg t_{\text{ox}}$. Since $l \gg l/m(Q)$, we may consider the microstrip line is infinitely long. From [9], the series impedance per unit length of the microstrip is given by

$$Z = \frac{1}{\sigma_m w_{\text{eq}} t_m} + \frac{j\omega\mu}{2\pi}\ln\left(\frac{2t_{\text{eq}}+(1-j)\delta}{r_{\text{eq}}}\right) \tag{8}$$

where σ_m is the microstrip conductor conductivity, $r_{\text{eq}} = (l/m(Q)+t_m)/4$, $t_{\text{eq}} = t_{\text{ox}} + (t_m - l/m(Q))/4$, and δ is the skin depth in the lossy silicon substrate given by

$$\delta = \frac{1}{\sqrt{\pi\mu f\sigma}} \tag{9}$$

where σ is the conductivity of the silicon substrate.

The characteristic impedance Z_0 of the equivalent microstrip line can be expressed from [10]

$$Z_0 = Z_{0\infty}\left(1+\frac{j\sigma}{4\pi f\varepsilon_0\varepsilon_{\text{Si}}}\right) \tag{10}$$

where

$$Z_{0\infty} = 120\pi\frac{1}{\sqrt{\varepsilon_{\text{so}}^{+}}}\frac{(t_{\text{ox}}+t_{\text{Si}})^{+}}{w_e} \tag{11}$$

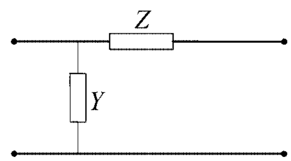

Fig. 3. Transmission-line model.

where the superscript “+”denotes effective values that take into account fringing and a mixed dielectric as follows:

$$\varepsilon_{\rm so}^{+} = \varepsilon_{\rm SiO}^{+}\frac{(t_{\rm ox}+t_{\rm Si})^{+}}{t_{\rm ox}^{+}} \tag{12}$$

$$\varepsilon_{\rm SiO}^{+} = \frac{1+\varepsilon_{\rm SiO}}{2} - \frac{1-\varepsilon_{\rm SiO}}{2}\left(1+10\frac{t_{\rm ox}}{w_e}\right)^{-1/2} \tag{13}$$

$$\frac{t^{+}}{w_e} = \begin{cases} \dfrac{1}{2\pi}\ln\left[\dfrac{8t}{w_e}+\dfrac{w_e}{4t}\right], & \text{for } \dfrac{t}{w_e}\geq 1 \\ \left[\dfrac{w_e}{t}+2.42-0.44\dfrac{t}{w_e}+\left(1-\frac{t}{w_e}\right)^{6}\right]^{-1}, & \text{for } \dfrac{t}{w_e}<1 \end{cases} \tag{14}$$

w_e is the equivalent width of the metal strip with a finite thickness, whose expression can be obtained from [11], [12] as

$$w_e = \begin{cases} w_{\rm eq}-\sigma_m+\dfrac{1.25t_m}{\pi}\left[1+\log\dfrac{4\pi(w_{\rm eq}-\sigma_m)}{t_m}\right], \\ \qquad \text{for } \dfrac{w_{\rm eq}-\sigma_m}{t_{\rm ox}+t_{\rm Si}}\leq\dfrac{1}{2\pi} \\ w_{\rm eq}-\sigma_m+\dfrac{1.25t_m}{\pi}\left[1+\log\dfrac{2(t_{\rm ox}+t_{\rm Si})}{t_m}\right], \\ \qquad \text{for } \dfrac{w_{\rm eq}-\sigma_m}{t_{\rm ox}+t_{\rm Si}}\geq\dfrac{1}{2\pi}. \end{cases} \tag{15}$$

By using the transmission-line model depicted in Fig. 3, the shunt admittance per unit length can be obtained by

$$Y = \frac{Z}{Z_0^2}. \tag{16}$$

The admittance matrix of the two-port network in Fig. 3 is

$$[Ym] = \frac{1}{Z}\begin{bmatrix} ZY+1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{17}$$

The quality factor of the spiral inductors is then obtained as follows:

$$Q_f = \frac{{\rm Im}\left(\dfrac{1}{Ym_{11}}\right)}{{\rm Re}\left(\dfrac{1}{Ym_{11}}\right)}. \tag{18}$$

III. Results and Discussion

The validity and accuracy of the DDM for parameters extraction of a spiral inductor are evaluated trough two examples by comparison with the experimental results. For a square spiral aluminum inductor with $N = 7$ turns, width $w = 13$ μm, and spacing between spires $s = 13$ μm, outer dimension $d_{\rm out} = 300$ μm, the spiral metal of three thickness values $t_m = 1, 2, 3$ μm are fabricated on a silicon substrate with resistivity $\rho = 10$ Ωcm and thickness $t_{\rm Si} = 150$ μm. The thickness of the SiO$_2$ film is $t_{\rm ox} = 4.5$ μm. The number of sub-domain of decomposition in the DDM is $n = 29$. The quality factor of the square spiral inductor with the three metal thickness values as a function of frequency is shown in Fig. 4 and compared with the experiment results in [2]. It is observed that there are approximately 10% deviations between the Q_f's peak value of the DDM model and measurement. The average relative error is 1.9%, 3.4%, and 3.8% for $t_m = 1, 2$ and 3 μm, respectively. As a second example, a circular spiral aluminum inductor on an Si–SiO$_2$ substrate with a conventional 0.6-μm CMOS three-metal-layer technology is considered. The inductors' geometrical characteristics are illuminated in the caption of Table I. The resistivity of the substrate is $\rho = 1 \sim 10$ Ωcm. The computed results from (18) are shown in Table I and are compared with the experiment results in [13]. The computed quality factor matches the experimental results with an average relative error of 4%, 2.8%, and 5.1%, respectively.

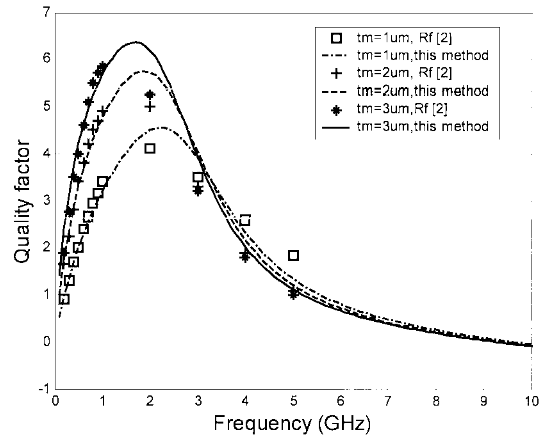

Fig. 4. Quality factor of the square inductor with difference metal thickness versus frequency.

The reason of the error includes: 1) the conformal mapping method is limited merely to quasi-static field analysis; 2) the equivalent transmission line in Fig. 1(c) is treated as of infinite length; and 3) the error of the DDM itself. According to (7), the maximum difference between the accurate total module of the square inductor and that from the DDM is

$$E = 4.41\left[\exp\big(-2\pi m(Q_1)\big)+\sum_{j=3}^{29}\exp\big(-2\pi m(Q_j)\big)\right] = 0.0262 \tag{19}$$

which means $315.7721 \leq m(Q) \leq 315.8245$, and will leads to a relative error less than 0.01% for the quality factor. This is to say that the DDM contributes very little to the computation error of the quality factor. The main error source should be the quasi-static approximation.

It is obvious from (19) that, in order to reduce the error of the DDM, all subdivisions of a domain should be as large as possible as long as the module of each subdivision can be computed easily. For the example, for a square spiral inductor, the

TABLE I
MEASURED AND SIMULATED RESULTS FOR THE CIRCULAR INDUCTOR

Ref.	Q Vs. Frequency								Q_{max}			Average relative error[%]
	1.272 [Ghz]		1.575 [Ghz]		1.86 [Ghz]		2.4 [Ghz]		Freq. [Ghz]	Value		
	Meas.	Sims.	Meas.	Sims.	Meas.	Sims.	Meas.	Sims.		Meas.	Sims.	
e_1	4	4.2	4.4	4.6	5	4.9	5.6	5.4	3	5.9	5.6	4
e_2	4.6	4.8	5.2	5.2	5.4	5.4	5	5.4	2	5.5	5.4	2.8
e_3	4.9	4.8	5.1	5.2	5.2	5.4	4.5	5.2	1.9	5.3	5.4	5.1

e_1: $d_{\text{in}} = 2 \times 75\ \mu$m, $w = 12\ \mu$m, $s = 1.5\ \mu$m, $N = 3$; e_2: $d_{\text{in}} = 2 \times 89\ \mu$m, $w = 12\ \mu$m, $s = 1.5\ \mu$m, $N = 4$; e_3: $d_{\text{in}} = 2 \times 90\ \mu$m, $w = 12\ \mu$m, $s = 1.5\ \mu$m, $N = 4$.

decomposition in Fig. 1(a) in which the crosscuts are set at every corner of the spiral is the best one. More subdivisions will have no help for improving the accuracy of the DDM. The situation of the circular spiral inductor is the same.

IV. CONCLUSION

Based on the DDM for computing conformal modules, a novel transmission-line model is derived for spiral inductors on multilayer substrate. The complicated computation of mutual parameters is avoided with the DDM. The model accuracy is analyzed, showing that the error due to the DDM is very little and can be neglected. This model can be extended to an arbitrary spiral inductor as long as one can solve for the conformal module of each subdivision in the decomposition of the inductor. Due to that, closed-form expressions are available for the quality factor, and the transmission-line model is expected to be efficient for analysis and optimization of RF circuits.

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